(7 pages) **Reg. No. :**.....

Code No. : 20572 E **Sub. Code : SMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics — Core

ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The direction cosines of the x-axis are _____.

   (a) $(1, 0, 0)$ (b) $(0, 0, 0)$

   (c) $(0, 1, 0)$ (d) $(0, 0, 1)$

2. If l, m, n are the direction cosines of a line, then

   (a) $l^2 + m^2 + n^2 = 1$ (b) $\frac{l}{m} = \frac{m}{n} = \frac{n}{l}$

   (c) $lm + mn + nl = 1$ (d) $l + m + n = 1$

3. Degree of a plane equation is _____.

(a) 4 (b) 3

(c) 2 (d) 1

4. The equation of the plane through $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

(a) $ax + by + cz = 0$ (b) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

(c) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ (d) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

5. The equation of the y-axis is

(a) $y = 0 ;\ z = 0$ (b) $y = 0$

(c) $x = 0 ;\ z = 0$ (d) $x = 0 ;\ y = 0$

6. $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r$ is the equation of the _____.

(a) circle (b) straight line

(c) ellipse (d) hyperbola

7. In the equation of the sphere, the coefficient of yz is

(a) 0 (b) 1

(c) 2 (d) 3

8. The diameter of the sphere $x^2 + y^2 + z^2 = 25$ is

(a) 10 (b) 25

(c) 50 (d) 5

9. Every line meets the cone in _____ points.

(a) 1 (b) 2

(c) 3 (d) 4

10. The axis of the cylinder is _____ to the generator of the cylinder.

(a) perpendicular (b) parallel

(c) equal (d) different

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the points $(10, 7, 0)$, $(6, 6, -1)$ and $(6, 9, -4)$ from an isosceles right angled triangle.

Or

(b) Find the angle between the lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2.

12. (a) Derive the equation of the plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

Or

(b) Find the equation of the plane passing through the points $(3, 1, 2)$ and $(3, 4, 4)$ and perpendicular to the plane $5x + y + 4z = 0$.

13. (a) Find the perpendicular distance from $P(3, 9, -1)$ to the line $\frac{x+8}{-8} = \frac{y-31}{1} = \frac{z-13}{5}$.

Or

(b) Find the condition for the lines $ax + by + cz + d = 0 = a_1x + b_1y + c_1z + d_1$, $a_2x + b_2y + c_2z + d_2 = 0 = a_3x + b_3y + c_3z + d_3$ to be the coplanar.

14. (a) Find the equation of the sphere which has its centre at the point $(6, -1, 2)$ and touches the plane $2x - y + 2z - 2 = 0$.

Or

(b) Show that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$.

15. (a) Show that the equation of a right circular cone whose vertex is 0, axis OZ and semi vertical angle α is $x^2 + y^2 = z^2 \tan^2 \alpha$.

Or

(b) Find the equation of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{27}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If the direction cosines of the two lines satisfy the equations $l + m + n = 0$; $2lm + 2\ln - mn = 0$; then find the angle between the lines.

Or

(b) A line make angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$.

17. (a) Find the equation of the plane which passes through the point $(-1, 3, 2)$ and perpendicular to the two planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$.

Or

(b) Find the equation of the plane through the point $(1, -2, 3)$ and the intersection of the planes $2x - y + 4z = 7$ and $x + 2y - 3z + 8 = 0$.

18. (a) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ and $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plane through them.

Or

(b) Find the shortest distance and equation of the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

19. (a) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

Or

(b) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$, $x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.

20. (a) Find the equation of the right circular cylinder described on the circle through the points $(a, 0, 0)$, $(0, a, 0)$, $(0, 0, a)$ as a guiding curve.

Or

(b) Find the condition for the plane $lx + my + nz = 0$ to touch the quadric cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

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